Beyond the Visual Impedance Effect

Alice Ping Ping Tse[1]**, Marco Ragni**[2]**, Johanna Lösch**[3]
[1]tse@tf.uni-freiburg.de, [2]ragni@tf.uni-freiburg.de, [3]j.losch@student.maastrichtuniversity.nl
Cognitive Computation Lab, Department of Computer Science, Georges-Köhler-Allee 52,
79110 Freiburg, Germany

Abstract

Whether the mental representation of reasoning problems is spatial or visual (or mixed) in nature has been the subject of considerable debate for years. The visual impedance effect found in Knauff & Johnson-Laird (2002) has provided us with new insights into this question. The study found that the forming of excessive visual images induced by the premises can impede relational reasoning. This study aimed at investigating the factor of complexity on the visual impedance effect in two folds, namely number of term series (i.e., total number of premises plus the conclusion) and whether the entities in the premises are presented in a continuous manner (i.e., whether the subject of the argument is the same as the object of the previous argument). In line with previous studies, relational category, number of term series and successiveness were the main factors of the response time. Results of the parameter estimation by generalized estimating equation showed that visual relations, 5-term series and discontinuous problems were the only significant parameters. The results again suggested that irrelevant visual images can hinder reasoning processes, in addition to the complexity of the problem. We proposed a combined cognitive model of ACT-R and PRISM for the findings in this study.

Keywords: Visual impedance effect; generalized estimating equation; PRISM; ACT-R.

Introduction

Reasoning is one of the complex cognitive processes which requires several fundamental processes and the role of working memory is irrefutable (e.g., García-Madruga et al., 2007; Barrouillet & Lecas, 1999). The multicomponent model of working memory proposed by Baddeley and colleagues is the most influential one. They proposed a model with four components, namely the central executive, the phonological loop, the visuospatial sketchpad and the episodic buffer (Baddeley & Hitch, 1974; Baddeley, 2000). The central executive acts as the managing unit which supervises the integration of information and coordinates the two "slave systems" (the phonological loop and visuospatial sketchpad, which act as short-term storage units for information) and other cognitive processes. The phonological loop stores phonological information while the visuospatial sketchpad stores visual and spatial information, which can be further divided into the visual subsystem (for visual information such as color and shape) and the spatial subsystem (for information related to location or space, e.g., Sima et al., 2013). This distinction is supported by the findings in behavioral experiments with a dual-task paradigm (that visual tasks were more hindered by a visual secondary task than a spatial one but spatial tasks by a spatial secondary task than a visual one (e.g., Klauer & Zhao, 2004)) as well as functional brain imaging studies (e.g., Smith et al., 1995).

There are proposals of individual differences in reasoning strategy that some people reason with a verbal and propositional while others reason with visuospatial representation (e.g., Ford, 1995; Bacon et al., 2007). However, the focus of this study will be on the representation of the problems instead of the strategy (see also the two-streams hypothesis – the ventral and dorsal streams for visual and spatial locational processing respectively (Goodale & Milner, 1992; Smith et al., 1995)). On the other hand, the presence of the two separate subsystems of the visuospatial sketchpad may suggest two possible representation formats of the reasoning problems during processing.

Some studies showed that the mental representation of reasoning problems can be either visual or spatial (Landau & Jackendoff, 1993; Huttenlocher, 1968). If the mental representation is visual, visual-rich materials should enhance the reasoning performance as some extra effort is required to represent visually opaque objects. However, many studies did not find this results (e.g., Sternberg, 1980). For example, similar behavioral performance was found for both abstract and concrete problems (Johnson-Laird et al., 1989; Newstead et al., 1986). However, the first study on visual images and relational reasoning by De Soto et al. (1965) found that the ease of visualizing the materials was a significant factor of the performance in their deductive reasoning experiment of "3-term series" problems:

> Ann is taller than Beth.
> Cath is shorter than Beth.
> Who is tallest?

Only the relations between A and B as well as B and C were given. Participants had to infer the relation between A and C from the two given relations. In other studies, participants were asked to evaluate the validity of the conclusion (e.g., Ann is taller than Cath) given the premises. The results were replicated in the studies by Shaver et al. (1975) and Clement & Falmagne (1986).

On the other hand, Knauff & Johnson-Laird (2002) found the exact opposite phenomenon – the visual impedance effect that excessive representations of visual information can slow down the reasoning process. They then concluded that representation of reasoning problems should be spatial in nature. Visual representations are assumed to be visual mental images which resemble the real object (Knauff & May, 2006); while spatial representations relate objects in spatial organization. By default, visual relational problems are represented visually while spatially for spatial problems after the comprehension process of the reasoning problems. Therefore, spatial problems are ready for reasoning processing if reasoning takes entities which are represented spatially. On the other hand, visual problems need an extra step to represent them spatially. Furthermore, the irrelevant visual information (to reasoning processing) can take up cognitive resources for and compete with the reasoning processes and thus causing an overall longer processing time. They argued that the opposite results in some previous studies were due to the neglect of the two possible different representations – visual and spatial: that some "visual" items were also "spatial". Therefore, the opposite results to the visual enhancement proposal were found when the target words of the relational problems were carefully controlled to visual (e.g., cleaner-dirtier: easy to visualize but difficult to form a spatial representation), visuospatial (e.g., above-below: easy to form spatial representation) and control (e.g., better-worse: difficult to visualize and form a spatial representation). They then revised the mental model theory[1] that mental models should be iconic spatial in nature, rather than visual (in addition to the fact that depicting abstract relations in a spatial manner is plausible and possible). The hypothesis was supported by further studies with blind people Knauff & May (2006) and on individual differences ("verbalizers" versus "visualizers" by Castañeda & Knauff, 2013).

Another source of reasoning difficulty depends on the number of mental model operations that are necessary to integrate information into a spatial model. The PRISM-model (Ragni & Knauff, 2013) formalizes these operations to build a preferred and alternative mental models (in cases which the descriptions allow for several models) and it provides a cognitive complexity measure[2] to predict and explain reasoning difficulty for model-based complexity.

This study aimed to investigate the influence of task complexity on the visual impedance effect in relational deductive reasoning tasks. We varied the task complexity in two ways. Firstly, besides the 3-term series problems, we included also the 4-term (3 premises) and 5-term series (4 premises) problems. Difficulty is expected to increase with the number of relations per task. Secondly, difficulty in terms of premises integration was varied. Discontinuous and continuous premise order problems (successiveness) were presented. For continuous problems, the subject of the premise sentence was the same as the object of the previous premise, except for the last premise preceding the conclusion, see Table 2 for examples. For continuous tasks, the information of the first premises can be integrated directly but in discontinuous tasks, the integration process cannot be proceeded until the presentation of the last premise. The information of previous premises has to be stored in the working memory for further processing. Therefore, problems presented in discontinuous order are expected to require more cognitive effort. We analyzed these three factors of difficulty/complexity (relational category, number of terms and successiveness) in terms of both the percentage of correct responses and response time. We hypothesized that only the number of terms and successiveness would affect the percentage of correct responses but not the relational category. However, all the three factors were expected to affect the response time as they can all slow down the reasoning processes. If the two complexity factors can affect the response time of visual relational problems, this might suggest that the underlying processes are the same, or at least involved.

The remainder of the article is structured as follows: In the following section, we present two experiments that investigate the visual impedance effect and model complexity. We then present a combined cognitive model in the next section. A general discussion concludes the article.

Experiments

One criticism of previous studies is that the target words in the experiments were not good examples of the categories, i.e., some "visual" items were also "spatial". In order to eliminate this potential limitation, we performed a preliminary survey to select two pairs of words for each of the four categories. We selected 37 pairs of antonyms (74 words) that may belong to the categories according to our intuition. They are all frequent words in daily usage. Two pairs of words for each category were selected to construct the reasoning problem set according to the results of the word survey.

Word Survey

Purpose of this word survey is to select the best two pairs of antonyms of adjectives describing the 1. visual, 2. spatial, 3. visuospatial, and 4. non-visual and non-spatial features of a noun. Knauff & Johnson-Laird (2002) have shown that the difficulty in forming spatial representation of words can be independent of that in forming visual images. Word pairs were then selected according to the criteria stated in Knauff & Johnson-Laird (2002).

Participants

The survey was performed on the Amazon Mechanical Turk platform. 97 native English speaker participated in the study (mean age = 36.8 years, SD = 13.48; 56 females). They received a nominal fee for their participation.

Materials and Design

We tested 37 pairs of antonyms such as fatter-thinner and below-above. Each trial consisted of one pair. Participants had to rate the ease of forming detailed visual images and spatial representation (position of the object in space) for each of the two words, i.e., giving four ratings in each trial. They were asked to move separate sliders for the two scales to indicate their responses, from 0 (extremely easy) to 100 (extremely difficult).

Results

We ranked the 74 words according to the means and medians of the two ratings (visual and spatial) respectively and

[1]The Mental Model Theory (e.g., Johnson-Laird & Byrne, 1991) proposes that reasoners apply three stages: premise comprehension, a subsequent integration to form an initial model, and conclusion validation by searching for counterexamples.

[2]The model: `http://spatialmentalmodels.appspot.com/`

Table 1: Mean rating for the ease of forming a visual image and a spatial representation for the two pairs of words.

	Word Pairs	Visual (mean)	Spatial (mean)
Visual	prettier–uglier	15.9	72.2
	cleaner–dirtier	15.3	67.7
Visuospatial	above–below	17.8	18.0
	in front of–behind	17.6	15.5
Spatial	right–left	25.4	17.6
	east–west	28.8	19.0
Control	kinder–crueler	42.0	68.8
	braver–more cowardly	43.6	70.7
Overall		29.7	51.7

Note. Range: 0 (extremely easy) to 100 (extremely difficult).

selected two pairs for each of the four categories. The two visual pairs are at the highest rank for the visual rating but are relatively low for the spatial rating. We could not find two pairs which were high for the spatial rating but very low for the visual rating and thus we chose two pairs which are ranked high for the spatial rating but not too high for the visual rating as the spatial pairs. The visuospatial pairs are at a high rank for both the visual and spatial ratings. We then chose two control pairs which were ranked relatively low for both the visual and spatial rating. Please refer to Table 1 for the ratings of the words we selected for each of the categories.

Reasoning Experiment

In this experiment, we investigated if the two other factors (complexity) of reasoning processing can affect reasoning performance and along with the visual impedance effect. The complexity was varied by increasing the number of premises (3-term, 4-term and 5-term series) and the successiveness of the entities in the problems (continuous or discontinuous). Visual relational category problems were expected to have the longest response time, then the control, and finally the visuospatial and spatial problems. 5-term discontinuous visual problems were expected to have the longest response time among the 24 possible condition combinations (4 category x 3 different term series x continuous or discontinuous).

Participants

96 native English speakers, different from the previous word survey, participated in the experiment (mean age = 36.35 years, SD = 11.96; 65 females) via the same platform. They received a nominal fee for their participation.

Materials and Design

We used the 8 pairs of words listed in Table 1 to construct 48 relational reasoning problems. We adopted a 4 (category: visual, spatial, visuospatial, and control) x 3 (term: 3-term, 4-term and 5-term) x 2 (continuous or discontinuous) x 2 (valid or invalid) design. All factors were within-subject as we were not interested in individual differences in this study. Therefore, there were 12 problems for each category and 16 problems for each term condition. We used the same animal-nouns for all the problems: ape, cat and dog for 3-term problems, ape, cat, dog and bird for 4-term problems and ape, cat, dog, bird and fish for 5-term problems. Half of the problems were continuous that the subject (the first noun) of the premise was the same as the object (last noun) of the previous premise, except for the last premise that either the subject or object was the same as the one in the previous premise (depending on the validity of the problem). Please refer to Table 2 for examples of the problems. The relational target word of the last premise was always the antonym of the previous premise(s). Each term in the antonym pair was presented equally often in the premises and conclusions. Each premise and the conclusion were presented on separate screens and the participants had to press the spacebar in order to read the next premise or the conclusion (offline self-paced design). The premises were presented in black letters while the conclusion was in red. Participants were asked to evaluate whether the given conclusion followed necessarily from the premises, by pressing the J (for yes) and D (for no) keys. Two practice trials were presented before the experiment. The relational target words (older-younger and faster-slower) in the practice trials were not repeated in the experiment. Reading time for each premise, response time to the conclusion and the response were recorded.

Results

Regarding the percentage of correct responses, the overall result was 69%. We have performed a 4 (category: visual, spatial, visuospatial and control) x 2 (term: 4-term or 5-term) x 2 (successiveness: continuous or discontinuous) x 2 (validity: valid and invalid) repeated measure ANOVA, excluding the 3-term problem (as there was no continuous and discontinuous distinction for the 3-term problems). The repeated measures ANOVA with a Greenhouse-Geisser correction showed that only successiveness and validity were significant main within-subject effects for percentage of correct responses, successiveness: $F(1,98) = 12.493, p = .001$, $\eta_p^2 = .113$; validity: $F(1,98) = 7.564, p = .007$, $\eta_p^2 = .072$. The post-hoc Wilcoxon Signed Ranks Test showed that valid problems had a significantly higher percentage of correct responses than invalid ones as well as continuous than discontinuous problems, $Z = 2.350, p = .019$ and $Z = 3.546, p < .001$, respectively. In line with previous studies, we did not find any effect due to the relational category of the target words. However, it was unexpected that the number of terms did not affect the percentage of correct responses.

Only response times for correct responses were included in the following analyses. We excluded also the response times which were not within $+/-$ 2 standard deviations of the mean response time of individual participant. Regarding the response time, we firstly tested if the validity was a significant factor of the response time. The Wilcoxon Signed Ranks Test showed that there was no significant difference

Table 2: Examples of 3-term and 5-term series. Conclusions can be valid (= v) or invalid (= i).

	3-term series	5-term series	
		Continuous order	Discontinuous order
Premise 1	The ape is dirtier than the cat	The bird is right of the ape	The ape is braver than the cat
Premise 2	The dog is cleaner than the cat	The ape is right of the fish	The dog is braver than the fish
Premise 3		The fish is right of the cat	The bird is braver than the ape
Premise 4		The dog is left of the cat	The dog is more cowardly than the cat
Conclusion	The ape is dirtier than the dog? (v)	The bird is right of the dog? (v)	The fish is braver than the bird? (i)

between the response time for valid and invalid problems, $Z = .837, p = .402$. Therefore, we aggregated these 2 conditions in order to simplify the following analyses. As there was no successiveness distinction for the 3-term condition, we performed a 4 (category: visual, spatial, visuospatial and control) x2 (term: 4-term or 5-term) x2 (successiveness: continuous or discontinuous) repeated measure ANOVA, excluding the 3-term problems. The repeated measures ANOVA with a Greenhouse-Geisser correction showed that category, term and successiveness were all significant main effects of response time, category: $F(1.899, 34.178) = 13.162, p < .001$, $\eta_p^2 = .422$; term: $F(1,18) = 36.680, p < .001$, $\eta_p^2 = .671$; successiveness: $F(1,18) = 21.032, p < .001$, $\eta_p^2 = .405$. However, none of the interaction effects were significant. We then performed a separate one-way repeated measure ANOVA with a Greenhouse-Geisser correction for the 3-term problems regarding the factor category. The results suggested that response times for different category conditions were significantly different, $F(2.316, 215.426) = 7.504, p < .001$, $\eta_p^2 = .075$. The response times are shown in Table 4.

As our aim was not to investigate individual differences, but rather factors on reasoning processing and related to visual impedance effect, all the aforementioned factors were within-subject factors. Regression analysis is not recommended for repeated measure data. We then used a generalized estimating equation (GEE) to estimate the parameters on response time. GEE was used to estimate the parameters of a generalized linear model with a possible unknown correlation between outcomes (Pickles, 1998) and is suitable for repeated measure data. GEE attempts to treat the within-subject covariance as a nuisance and model only the mean response. In additional, it is not necessary to specify the covariance structure correctly for reasonable estimates of regression coefficients and standard errors as it is not modeled and thus is relatively more flexible. GEE was used to analyze the data with response time as the dependent variable and category, term and successiveness as the factors. In order to simplify the model, only the 3 main effects were included, as the results of the repeated measure ANOVA suggested no significant interaction effects. The model suggested that for the response time $RT = 112118 + 2224 \cdot visual + 2354 \cdot (5 - term) + 1145 \cdot discontinous$. Visual, 5-term and discontinuous problems required longer processing time with 5-term and visual required about the double time than discontinuous problems. We could actually hypothesize the relative response time for the problems of each of the condition combinations according to the equation, but space in this article does not permit an extensive discussion here. Please refer to Table 3 for the results.

A Cognitive Model

We investigated in one experiment two different often replicated effects: The (dis-)continuity effect (successiveness) and the visual impedance effect that both have an implication on the reasoning time. While the former can be explained by the PRISM-model (Ragni & Knauff, 2013), the later has been recently explained by a model of memory effect in ACT-R (Albrecht et al., 2015). PRISM is a computational cognitive model that can be used to simulate and explain how spatial mental models are constructed, inspected, and varied in a spatial array that functions as if it were a spatial working memory. A spatial focus inserts tokens into the array, inspects the array to find new spatial relations, and relocates tokens in the array to generate alternative models of the problem description, if necessary. Each of these focus operations in PRISM imply 1 standard cost (see website in footnote 2), defining a cognitive complexity measure. It does not reflect, however, how visual or spatial the relations are – it calculates for each problem the costs necessary for performing the associated mental model operations to reason about this problem.

While the ACT-R model of memory effect does not necessarily reflect mental model operations, it is compatible with PRISM (Albrecht et al., 2015). The cost function for PRISM, combined with an additional costs for a visual representation (1 standard unit) and a reduction cost of -0.5 standard unit for spatial or visuospatial representation compared to the control problems (as the spatial representation is already built during the comprehension process), can be expressed by the following cost function:

$$cost(x) = 5 + 1 \cdot visual - 0.5 \cdot visuospatial - 0.5 \cdot spatial + 2 \cdot (\#terms - 3) + 2 \cdot discontinuous$$

with four boolean variables (i.e., 1 if *true* else 0) *visual*, *visuospatial*, *spatial*, and *discontinuous*. *#terms* represent the number of terms in the problem. While we are working on a computational cognitive modeling incorporating the cognitive complexity measure in the formula, the correlation between the respective behavioral response times and values (costs) calculated by the cost function is very high (Kendall's $\tau = .974$, $p < .001$). This makes the effects on building and

Table 3: Results of the parameter estimation by the Generalized Estimating Equations.

Parameter Estimates							
Parameter	β	Std. Err.	95% Wald CI		Hypothesis Test		
			Lower	Upper	Wald χ^2	df	p-value
(Intercept)	12051	669	10739	13363	324.172	1	.000
Visuospatial	338	631	-898	1574	.287	1	.592
Visual	2633	772	1121	4146	11.647	1	.001
Spatial	-21	355	-718	676	.003	1	.953
Control	Redundant						
5-Term	2482	446	1607	3356	30.948	1	.000
4-Term	Redundant						
discontinuous	1151	338	488	1814	11.593	1	.001
continuous	Redundant						
(Scale)	123440405						

Note. β = beta coefficient; Std. Err. = Standard Error; Wald CI = Wald confidence interval.

Table 4: Mean response times (in ms) for correctly answered problem. Response times not within +/- 2 standard deviations of the mean time of individual participant were excluded. Cont = continuous; disc = discontinuous.

	3-term	4-term		5-term	
Category		cont	disc	cont	disc
Control	9727	13791	15490	16888	18156
Spatial	9056	12254	16140	16406	17723
Visual	10972	15219	17556	19765	21016
Visuospatial	8948	12833	16295	15352	17484

keeping visual models and cognitive operations comparable on the common ground of mental operations.

General Discussion

Knauff & Johnson-Laird (2002) suggested that visually-rich materials can evoke extra cognitive loading due to the processing of unnecessary visual information in reasoning which causes the visual impedance effect; in contrast with the common belief that reasoning problems with materials which are easier to be visualized are easier to solve, having a faster processing speed. Representation of reasoning problems should be spatial rather than visual according to the results. Otherwise, visual-relational problems should be faster to solve if the problems are represented visually. In line with the results of Knauff & Johnson-Laird (2002), we found a slower response time for visual problems even when the complexity of the problems was varied. The same trend was found in 3-term series as well as 4-term and 5-term series for both continuous and discontinuous problems. Moreover, we found that the number of term and successiveness of the entities in the problems were also factors of the response time. Only successiveness and validity were the significant factors for the percentage of correct responses. However, validity does not affect the response time of the problem.

It might be possible that the phenomenon is due to more spreading activation in memory of more (visual) features for visually-rich relational terms Albrecht et al. (2015). As more linked features are activated, the access time is longer. However, we found a stronger visual impedance effect for 5-term discontinuous problem than 5-term continuous problems (with Wilcoxon Signed Ranks Test, difference between response time for visual and control problems were significant, for continuous: $Z = 2.262, p = .024$; for discontinuous: $Z = 2.809, p = .005$). The impedance effect is quite possibly affecting the same underlying processing, i.e., hindrance in reasoning processing. If the more spreading of activation in memory solely causes the visual impedance effect, discontinuous problems should not have an even slower response time as the items in both kinds of problems were the same and they should evoke similar activations in memory.

As suggested by the mental model theory, after representing the premises into a iconic spatial array with the semantics process, where the visual impedance effect is induced, the reasoning processes (i.e., the construction of iconic spatial mental model(s), validity evaluation, and revision of the initial model) are nonlinguistic in nature. This hypothesis was reflected in our results. We did not find any difference between percentage of correct responses for visual relational problems and other relational categories. On the other hand, successiveness and validity were significant factors. Invalid problems had a lower percentage of correct responses as well as discontinuous problems. It was out of our expectation that 5-term series problems were not more difficult. The difference between 4-term and 5-term series problems was very small (4-term: 67%; and 5-term: 65%). The results suggested that the ease in representing or forming the integrated representation is a more important issue in reasoning.

Results of the GEE analysis were in line with those of the repeated measure ANOVA of the response time. Both results suggested that 5-term series discontinuous visual problems had the slowest response time. Furthermore, the regression coefficient of the visual problems was biggest among the three significant parameters which might suggest that the impedance due to visual processing was a more important factor for the impedance effect on reasoning than the number of premises and successiveness of the entities.

In conclusion, our study replicated the visual impedance effect for problems with different complexity. We found that visual relational terms can slow down reasoning processing but spatial relations can enhance reasoning in some of our conditions. This supported the hypothesis of Knauff & Johnson-Laird (2002) that the mental representation of reasoning problems should be spatial in nature. Also, the factors affecting the complexity of reasoning problems are important for the response time. Limitations of our study include that we could not find any spatial relation terms that are easy to represent spatially but difficult to visualize. In our word survey, we found many pairs which are easy to be visualized but only few are rated to be easy to envisage spatially. However, this is one of the confounding factors of many previous studies. Further studies include the correlation between the easiness in visualizing the problem and the strength of the visual impedance effect. The effect can also be tested with other kinds of reasoning problems such as syllogisms.

Acknowledgement

Supported by DFG-projects RA 1934-2/1 and RA 1934-3/1.

References

Albrecht, R., Schultheis, H., & Fu, W.-T. (2015). Visuospatial memory processing and the visual impedance effect. In D. C. Noelle et al. (Eds.), *Proceedings of the 37th Annual Meeting of the Cognitive Science Society.* Pasadena, California: Austin, TX: Cognitive Science Society.

Bacon, A. M., Handley, S. J., & McDonald, E. L. (2007). Reasoning and dyslexia: A spatial strategy may impede reasoning with visually rich information. *British Journal of Psychology*, *98*(1), 79–92.

Baddeley, A. D. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences*, *4*(11), 417–423.

Baddeley, A. D., & Hitch, G. (1974). Working memory. *Psychology of learning and motivation*, *8*, 47–89.

Barrouillet, P., & Lecas, J.-F. (1999). Mental models in conditional reasoning and working memory. *Thinking & Reasoning*, *5*(4), 289–302.

Castañeda, L. E. G., & Knauff, M. (2013). Individual Differences, Imagery and the Visual Impedance Effect. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceedings of the 35th Annual Meeting of the Cognitive Science Society.* Berlin, Germany.

Clement, C. A., & Falmagne, R. J. (1986). Logical reasoning, world knowledge, and mental imagery: Interconnections in cognitive processes. *Memory & Cognition*, *14*(4), 299–307.

De Soto, C. B., London, M., & Handel, S. (1965). Social reasoning and spatial paralogic. *Journal of Personality and Social Psychology*, *2*(4), 513.

Ford, M. (1995). Two modes of mental representation and problem solution in syllogistic reasoning. *Cognition*, *54*(1), 1–71.

García-Madruga, J. A., Gutiérrez, F., Carriedo, N., Luzón, J. M., & Vila, J. O. (2007). Mental models in propositional reasoning and working memory's central executive. *Thinking & reasoning*, *13*(4), 370–393.

Goodale, M. A., & Milner, A. D. (1992). Separate visual pathways for perception and action. *Trends in Neurosciences*, *15*(1), 20–25.

Huttenlocher, J. (1968). Constructing spatial images: A strategy in reasoning. *Psychological Review*, *75*(6), 550.

Johnson-Laird, P. N., & Byrne, R. M. (1991). *Deduction*. Lawrence Erlbaum Associates, Inc.

Johnson-Laird, P. N., Byrne, R. M., & Tabossi, P. (1989). Reasoning by model: The case of multiple quantification. *Psychological Review*, *96*(4), 658.

Klauer, K. C., & Zhao, Z. (2004). Double dissociations in visual and spatial short-term memory. *Journal of Experimental Psychology: General*, *133*(3), 355.

Knauff, M., & Johnson-Laird, P. (2002). Visual imagery can impede reasoning. *Memory & Cognition*, *30*(3), 363–371.

Knauff, M., & May, E. (2006). Mental imagery, reasoning, and blindness. *The Quarterly Journal of Experimental Psychology*, *59*(1), 161–177.

Landau, B., & Jackendoff, R. (1993). Whence and whither in spatial language and spatial cognition? *Behavioral and Brain Sciences*, *16*(02), 255–265.

Newstead, S. E., Pollard, P., & Griggs, R. A. (1986). Response bias in relational reasoning. *Bulletin of the Psychonomic Society*, *24*(2), 95–98.

Pickles, A. (1998). Generalized estimating equations. *Encyclopedia of biostatistics*.

Ragni, M., & Knauff, M. (2013). A theory and a computational model of spatial reasoning with preferred mental models. *Psychological Review*, *120*(3), 561.

Shaver, P., Pierson, L., & Lang, S. (1975). Converging evidence for the functional significance of imagery in problem solving. *Cognition*, *3*(4), 359–375.

Sima, J. F., Schultheis, H., & Barkowsky, T. (2013). Differences between spatial and visual mental representations. *Frontiers in psychology*, *4*.

Smith, E. E., Jonides, J., Koeppe, R. A., Awh, E., Schumacher, E. H., & Minoshima, S. (1995). Spatial versus object working memory: PET investigations. *Journal of Cognitive Neuroscience*, *7*(3), 337–356.

Sternberg, R. J. (1980). Representation and process in linear syllogistic reasoning. *Journal of Experimental Psychology: General*, *109*(2), 119.