Cognitive Computational Models for Conditional Reasoning

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Abstract

Premises in conditional reasoning consist of an "if" statement (e.g., "if I can catch the bus, I won't be late") and a fact (e.g., I can catch the bus). Such types of simple inference have been studied empirically and formally for about a century. In the past five decades, several cognitive theories have been proposed to explain why humans deviate from predictions of conditional logic. In this article, we (i) describe existing theories, (ii) develop multinomial processing tree (MPT) models for these theories and systematically extend the theories with guessing subtrees to test the predictive power of the cognitive models. The models are evaluated with G^2, Akaike's (AIC) and Bayesian Information Criteria (BIC), and Fisher's Information Approximation (FIA). Mental model theory with directionality for indicative conditionals while the independence model for counterfactuals provide the best fits to data from psychological studies.

Keywords: Human conditional reasoning; multinomial process trees; cognitive theories

Introduction

Suppositional and hypothetical thinking are one of the major cognitive abilities distinguishing humans from other animals. This form of thinking is essential to reflect on past events, hypothesize alternative outcomes, and partially prevent future mistakes. It also facilitates us to make and test assumptions about future outcomes to select actions, responses, precautions or/and procedures. This kind of thought is usually presented as conditional statements in natural language. A conditional statement is usually in the form of "if p then q", expressing a relationship between the *antecedent* p and a *consequent* q. Classical studies of reasoning always use sets of arguments consisting of a conditional and an additional categorical information ("a fact"), i.e., p, $\neg p$, q, $\neg q$. Consider the following problem:

> If I can catch the bus, I won't be late. (*conditional*)
> I can catch the bus. (*categorical*)
> What, if anything, follows?

Almost all reasoners draw "I won't be late" as a conclusion of the two statements. This is an example of a *modus ponens* (MP for short) inference, i.e., to conclude the consequent q (*I won't be late*) from the conditional and the categorical statement p (*I can catch the bus*). Other inference schemas are *modus tollens* (MT for short), i.e., to conclude $\neg p$ (*I cannot catch the bus*) from the conditional and an additional categorical statement $\neg q$ (*I will be late*).

Both schemas MP and MT are classically logically valid. The other two schemas, namely *denial of the antecedent* (DA, to conclude q when $\neg p$ is given as the additional categorical statement) and *affirmation of the consequent* (AC, to conclude p when q is given), are logically invalid but commonly drawn by humans. We focus on deductive reasoning in this article. While the classical logical interpretation is the so-called material implication (if the antecedent is true, the consequence cannot be false) and is easy to define, many psychological experiments have demonstrated that humans deviate from this interpretation. For example, conditional statements in subjunctive grammatical mood (i.e., counterfactual statements) can trigger a different endorsement pattern of the inferences (Byrne & Tasso, 1999; Thompson & Byrne, 2002), compared to statements in indicative mood, i.e., factual statements. It was found that people make inferences from counterfactual conditionals that are less frequently made, for example, when they are asked to reason from the two conditionals: 'If George kept his stock in Company B, then it earned $1,200 (Byrne & McEleney, 2000)' (factual) and 'If George had kept his stock in Company B, then he would have been better off by $1,200' (counterfactual). The two negative inferences, namely Modus Tollens (MT) and Denial of Antecedent (DA), had higher endorsement rates in the counterfactual than in the factual condition. We analyze different psychological theories while combining them with an idea from signal detection theory (Macmillan & Creelman, 2004). In visual perception (or memory recognition), the application is to test if humans can correctly identify or not the presence or absence of stimulus in an environment with background noise. We apply this idea to conditional reasoning as follows:

Inference does logically	Response of Ss "not follow"	Response of Ss "follow"
follow	miss	hit
not follow	correct rejection	false alarm

Both inference rules MP and MT are in the category of *logically follows*. Hence if they are applied, we have a *hit* (otherwise, we have a *miss*). If the inference rules AC and DA are not applied, we have a *correct rejection* (but a *false alarm* if applied). Oberauer (2006) has already formalized some theories with multinomial process trees (MPTs) for all the 16 possible answer patterns that are subsets of the four inferences. Hence, his tree included all cognitive processes altogether that led from an input to the 16 leaves which represent the responses. A single fixed guessing tree was inserted to each tree. The models were evaluated by G^2 (see later section for details).

Inspired by the aforementioned idea, we have systematically developed trees for each of the four inference patterns combined with parametrized guessing trees, determining different modes of guessing. That means instead of one tree for all the four inferences, 4 separate trees were constructed for each of MP MT AC and DA according to different cognitive theories. The remainder of this paper is structured as follows: In the next section, we will briefly review current existing theories for conditional reasoning. Then, we will represent these theories as multinomial process trees and systematically vary the amount of guessing for different theories. Then, we will review and report the model fitting results of 45 behavioral experiments (total number of participants $N = 2530$, datasets with the endorsement percentages and N provided for all the four inference rules) on conditional reasoning for simple/classical/indicative conditionals and 12 experimental datasets for counterfactual reasoning, $N = 577$. The cognitive theories formulated as multinomial process theories are then evaluated based on model selection criteria measures – the information criteria AIC and BIC which take additionally the model size into account. A discussion of the best cognitive theory in terms of predictive power concludes the paper.

Cognitive Models of Conditional Reasoning

We introduce some formal notations that we will use in the following sections. A conditional ("if p then q") is written as $p \to q$ or $(q \mid p)$. Negating a fact p is represented as $\neg p$, the same applies for q. Theories of conditional reasoning can be vastly classified into model-based, e.g., the theory of mental models (Johnson-Laird & Byrne, 1991), rule-based, e.g., the theory of mental logic (Rips, 1994), and theories that build on the idea of Bayesian modeling (Oaksford et al., 2000).

Theory of mental models

The mental model theory (MMT) of conditional reasoning (Byrne & Johnson-Laird, 2009) assumes that for a conditional $p \to q$, the semantic information of each premise is represented in an initial mental model akin to:

$$\begin{array}{c} p\ q \\ \ldots \end{array}$$

Hence both the antecedent and consequent are true in the initial mental model. If p is given, a modus ponens inference, can be drawn and q is derived. In cases where other information is given, e.g., $\neg q$, the model needs to be fleshed out, i.e., other true interpretations of the conditional need to be generated. This leads to the construction of three models eventually:

$$\begin{array}{rrl} p & q & \text{(initial mental model)} \\ \neg p & q & \text{(alternative mental model 1)} \\ \neg p & \neg q & \text{(alternative mental model 2)} \end{array}$$

Hence, an MP-inference is easy, while MT requires more cognitive effort to generate alternative models. MMT explains deviation of human reasoners from the normative logically correct performance by inaction or failure in the search of counterexamples and fleshing out of the initial mental model. The mental model theory does not make any assumption about the directionality of the antecedent and consequent. However, several studies have shown that the directionality of conditionals plays a role in the reasoning process (Evans & Beck, 1981; Barrouillet et al., 2000). We thus include both the classical and extended mental model theory by introducing the assumption about directionality (Oberauer, 2006).

The theory of mental logic

The mental logic theory suggests that humans translate the premises into logical form and use formal rules to draw or prove the conclusion (Rips, 1994; Braine & O'Brien, 1998). However, only MP and MT can be proved by formal rules. MP can be drawn directly with the formal rule of inference but the proof of MT requires several more steps, with reductio ad absurdum (finding a contradiction to the supposition of p). That means, reasoners firstly suppose p after reading the two premises and then find that the conclusion q (by applying the MP inference rule on the supposition) and $\neg q$ (the second/minor premise) are incompatible and thus reject the supposition of p using *reductio ad absurdum* and finally conclude $\neg p$. Errors in reasoning performance are due to misunderstanding of the conditional statements or the application of a wrong rule. As the endorsement of AC and DA rules cannot be explained by mental logic, we use guessing trees for these two inferences. It follows that implementing the mental logic as an MPT is not possible without any guessing trees.

Probabilistic approach: The independence model

Oaksford & Chater (1994) proposed a Bayesian understanding and modeling about how people interpret a conditional and reason about it. Instead of interpreting $p \to q$ in the classical logical sense – as material implication – human reasoners and their reasoning processes can be modeled as the conditional probability of q given p, i.e., $P(q \mid p)$. In their classical work, they proposed a dependence and an independence model. We focus on the later: the classical independence model (Oaksford & Chater, 1994) consists of two parameters a for $P(p)$ and b for $P(q \mid \neg p)$. To fit experiments, the best parameter values were determined by iterating through the values 0.1, 0.3, 0.5, 0.7, and 0.9 for both a and b as in Table 1 of Oaksford & Chater (1994). The model accepts a specific conditional probability only if the computed value is above a given threshold. We present here an updated version (Oaksford et al., 2000; Singmann et al., 2016). The model assumes that reasoning is done through assessing the probability values of conclusions based on the reasoner's background knowledge. More precisely, when asked to evaluate an inference such as MP, "Given 'If p then q' and 'p', how likely is q?", individuals consult their background knowledge regarding p and q and assess the conditional probability of the conclusion q given the minor premise p. Thus, endorsement E is modeled as $E(\mathsf{MP}) = P(q \mid p)$. The joint probability distribution of p and q, and their negations $\neg p$ and $\neg q$ can be parameterized in terms of three parameters, $a = P(p)$; $b = P(q)$,

Table 1: Oaksford et al. (2000) model of probabilistic conditional reasoning (see, Singmann et al., 2016).

	q	$\neg q$
p	$a \cdot (1-e)$	$a \cdot e$
$\neg p$	$b - a(1-e)$	$(1-b) - ae$

Note. The table represents the joint probability distribution for a conditional, "if p then q" by three parameters: $a = P(p)$, $b = P(q)$, and $e = P(\neg q \mid p)$.

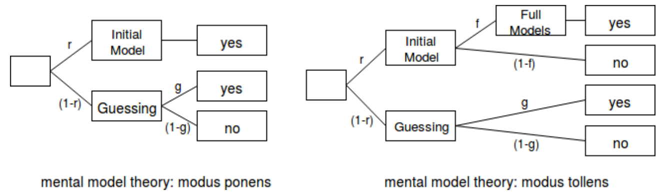

Figure 1: Two examples for MPTs for the mental model theory (without directionality). The left tree represents the model for the modus ponens; while the right represents the model for the modus tollens, where an additional flesh-out process from the initial mental model is necessary.

and $e = P(\neg q \mid p)$ as shown in Table 1, which leads to the following model predictions (cp. Singmann et al., 2016):

$$E(\mathsf{MP}) = P(q \mid p) = (1-e) \tag{1}$$

$$E(\mathsf{MT}) = P(\neg p \mid \neg q) = \frac{1-b-ae}{1-b} \tag{2}$$

$$E(\mathsf{AC}) = P(p \mid q) = \frac{a(1-e)}{b} \tag{3}$$

$$E(\mathsf{DA}) = P(\neg q \mid \neg p) = \frac{1-b-ae}{1-a} \tag{4}$$

Many experiments, however, provided reasoners with premise information that they were asked to consider as *true*. These formulae can thus be reduced for our problems – the probability of the given fact in the experiments can be assigned as 1. Hence, it holds for the example $P(\text{"I catch the bus"}) = 1$. We can then represent the simplified independence model and transform it into an MPT. Consider modus ponens, with $P(p) = 1$. As $a = P(p)$, we have $a = 1$. Hence, the formula is reduced to $1-e$. For MT, $P(\neg q) = 1$, it follows $P(q) = 0$, and hence $b = 0$. Thus, (2) above is reduced to $(1-ae)$. For $E(\mathsf{AC})$ for AC with $b = P(q) = 1$, Equation (3) is reduced to $a(1-e)$. For $E(\mathsf{DA})$ holds, $a = P(p) = 0$, and (4) above is reduced to $(1-b)$.

The suppositional theory

The suppositional theory proposed by Evans & Over (2004) is a hybrid theory with the application of probability assumption akin to the independence theory and dual process theory, together with some rules in the field of pragmatics. Similar to the independence theory, it emphasizes the cases where we have a high probability of the consequent given the probability of the antecedent. Contextual effect found in a vast amount of studies in conditional reasoning can be explained by pragmatic inferences. Finally, the theory has a dual system incorporated: While immediate inferences (System 1) are solely drawn by the probability account, System 2 inferences are possible and lead to deductively valid answers (cp. Oberauer, 2006).

Theories of Conditional Reasoning as MPTs

Our main goal is to assess the empirical adequacy of the aforementioned cognitive theories. Towards this goal, we need to represent the theories formally. Following a similar approach by Oberauer (2006), we formalize the theories as *multinomial processing tree* (MPT) models (Riefer & Batchelder, 1988).

MPT models are a class of cognitive models for categorical data that describe observed response frequencies resulting from latent cognitive states. The probabilities that are represented at the edges in the graph for transitioning a cognitive states are estimated from data. At the same time, the aforementioned cognitive theories must explain why the answers of the participants often deviate from the logically correct solution as well. There are two ways of how responses are generated by a reasoner: a *reasoning process*, the process is described and/or predicted by a cognitive theory and a *guessing process*, a process that is not explained in a cognitive theory and, in principle, any possible response can be given.

We represent the reasoning and guessing parts of the theories by multinomial process trees as we outline in the following. For each of the four inference schemas (MP, MT, DA, AC) we develop separate MPT trees. As the theories assume different cognitive processes for the four inferences, we model them as four MPTs. An additional advantage is that this enables us to investigate the differences in processing of the four inferences, see Fig. 1 for an example for MP and MT for the mental model theory. Each root node contains a reasoning parameter r that is responsible for estimating responses that are generated by the reasoning subtree and consequently $(1-r)$ as generated by the guessing subtree. The guessing tree is identical for all theories, with a parameter g to guess a yes-answer and $(1-g)$ a no-answer. In the reasoning tree, we have theory specific nodes that provide specific answers by transitioning through them. In the case of modus ponens (the most simple case), the correct answer can be read out via going along the reasoning branch, where an initial model is built (the model $p \;\; q$). In the case of modus tollens, a full explicit model needs to be built (parameter f) for a correct answer and if it is not built $(1-f)$, the decision is solely made by the initial mental model. Hence, these process models do reflect assumed cognitive processes and they are similar enough to formalizations as proposed for syllogistic reasoning (cp. Klauer et al., 2000). In the following sections, we will investigate all previously mentioned theories: The mental model theory with and without directionality, the mental logic theory, the probabilistic theory and the suppositional theory – formalized as MPTs and the respective extended models with a reasoning part and a guessing tree (as described above). we systematically replace the reasoning parts by pure guessing trees (we will describe in detail later in the section "MPT analysis for model comparison"). We can assume that some reasoning subtrees may even have a negative impact, so we systematically eliminate for each theory the reasoning subtree in the extended models.

The Experimental Data

Selection of the experimental studies

We searched the literature and the internet for articles on classic conditional reasoning and non-monotonic conditional reasoning and reporting at least the number of participants as well as absolute number of reasoners or percentages for all four inference types (MP, MT, DA, AC). Extensive search of studies in Pubmed, Science Direct, Google and Google Scholar with the keywords "(conditional reasoning) or (conditionals) or (prepositional reasoning) or (counterfactual) or (alternatives) or (enabler) or (disabler)" was performed. Most of the articles are not suitable for this analysis as the endorsement percentages/frequencies of the four inference rules were not provided. We have included all experiments reporting the four inference types as within subject factor and the questions presented to participants was in the form of "what (if anything) follows (necessarily)?" or "think about what conclusion you can draw from the information" or "assess whether these conclusions follow logically from the information" or "Therefore, "; with two to four answer options provided to participants. For MP and DA, the answer options were "q", "$\neg q$" (and "may or may not q" and "not sure" or "nothing can be concluded"); and "p", "$\neg p$" (and "may or may not p", and "not sure" or "not nothing can be concluded") for MT and AC. We have excluded 3 experiments with special manipulation of the content of the conditional statements. This selection eliminates as possible the factors due to experimental design. Finally, 16 studies of indicative conditionals (first premise being in the form of "If p then q"; 45 experiments in total) and 6 studies of counterfactual conditional reasoning (12 experiments in total) of adult data were included[1]. We need the frequencies of participants endorsing each inference for our later analysis. For studies providing the endorsement percentages, we computed the frequencies by the percentages and the number of participants.

Reliability of data

We assessed the overall homogeneity for each inference by examining the respective rank orders of the endorsed inferences using Kendall's coefficient of concordance (W), which ranges from 0, no consensus, to 1, perfect consensus. The datasets are rather homogeneous for both indicative conditionals and counterfactuals, $W = .617$, $p < .001$ and $W = .767$, $p < .001$, respectively.

[1] For studies and MPTs, see: www.cc.uni-freiburg.de/data

Conditional reasoning with counterfactuals

Most of the studies on counterfactual reasoning were carried out by Byrne and colleagues (Byrne & Tasso, 1999; Thompson & Byrne, 2002). Usually, conditional statement in subjunctive mood (for native alphabetic languages speakers) were presented to participants to indicate the counterfactual (unreal) property of the situation described in the statement. In these studies, the two negative inferences, DA and MT, usually had a much higher endorsement percentages in the counterfactual than factual condition (but the endorsement percentages of the two positive inferences remained statistically the same). The results support the hypothesis of Byrne and colleagues that reasoners consider two alternatives when they encounter such counterfactual arguments, namely the fact and supposed "fact" (also known as the "presupposed factual reality" and "counterfactual conjecture"). Reasoners constructed already the following two models as the initial mental model and thus the two negative inferences are more likely to be drawn:

$p \quad q$	(counterfactual conjecture)
$\neg p \; \neg q$	(presupposed factual reality)

Besides, there are three other proposals applicable to counterfactual reasoning. For example, Lewis and Stalnaker's possible world semantics of modal logic (Lewis, 2013; Stalnaker, 1968). They proposed that reasoners assume another world which is most similar to the real world. They perform counterfactual reasoning through reasoning about this most-similar world. However, many researchers criticized the assumption that ordinary people do not judge the closeness of the world/possibility. We have only fitted the models of the adapted mental model theory for counterfactuals (both with and without directionality) as this theory explicitly makes assumptions about cognitive processes in human reasoning.

MPT Analysis for Model Comparison

We fitted each model to the aggregated data via the maximum likelihood method using `MPTinR` (Singmann & Kellen, 2013). The package uses four measures, and the smaller their values, the better the fit between a model and the data: First, G^2 measures the goodness of fit using the maximum-likelihood method, which maximizes the likelihood of the frequencies of observations given the parameter values. It underlies the remaining three measures. Second, the *Akaike information criterion* (AIC) indicates how much information is lost when a model represents the process that generates the data, taking into account both its goodness of fit and number of parameters. Third, the *Bayesian information criterion* (BIC) is a Bayesian analog of AIC that selects the best model from a finite set of them, penalizing models according to the number of their parameters. Fourth, the *Fisher information approximation* (FIA) measures the amount of information that an observed frequency carries about a parameter which models the observation. It provides a good measure of the flexibility of a cognitive model. We evaluated the five cognitive theories and some adaptation of the theories systemically. Firstly, we compared the MPT implementations of different cognitive theories (the original version) with only one guessing trees extension at the root node, see the $(1-r)$-paths in Fig. 1. Secondly, we systematically eliminated the reasoning subtree, the r-path in 1, 2, or 3 of the inference schemas MT, DA, and AC and kept the guessing tree only. If we replaced the reasoning subtree for the MT we denote it as Guess2. If we replace the DA and AC reasoning subtree, we denote it as Guess34. The reason is to investigate the positive or negative impact of the reasoning tree. The MP reasoning tree is never replaced by a guessing tree as most humans do not have difficulty in drawing MP inference and the sole use of guessing would thus be unnecessary and redundant. We use a PureGuess model which exclusively consists of guessing trees (no reasoning part in any of the four inference schemas) as the base line. The impact of the inference part and how it may disguise the processes can then be evaluated – by comparing if the reasoning part of the theories may add something substantially or not in the information criteria. We repeat the two analysis steps with datasets from counterfactual reasoning to check if the best models for indicative conditionals apply to counterfactual reasoning too.

Theory evaluation

Our first analysis deals with testing the predictive power of the aforementioned cognitive theories for human conditional reasoning. Table 2 reports the results of the four theories (excluding the mental logic as it only makes predictions for MP and MT) and additionally the pure guessing model PureGuess. The lower the G^2, AIC, BIC, and FIA the better the models are. Table 2 shows that the theory (extended with a guessing subtree) which fits best the data is the mental model theory with directionality, which differs from the suppositional theory in a better FIA. The PureGuess model performs worst, i.e., this shows that the reasoning parts of the theories contribute in explaining the data considerably.

Table 2: Results of MPT fits to the aggregated data set of classical conditionals, original version

Model	No. of parameters	G^2	FIA	AIC	BIC
MMTd	4	16.6	3	25	53
SUP	4	16.6	21	25	53
MMT	3	139.5	81	146	167
IND	3	492.5	*	499	520
PureGuess	1	653.1	331	655	662

Note. SUP = suppositional theory; MMTd = mental model theory with directionality; MMT = mental model theory; IND = independence model. PureGuess = pure guessing trees for MP, MT, AC, and DA. * The independence model is not a binary MPT so FIA cannot be computed.

Impact of guessing

In the next analysis, we investigated what happens if we systematically eliminate inference parts according to the theories. Table 3 reports the 5 best fitting theories out of 34 theories. Except the mental logic (with only 2 variants: ML-Guess34 and ML-Guess234), all the other 4 theories have 8 variants (total = 4*8 + 2 = 34). The models are ordered regarding the best values for the information criteria BIC, AIC, and FIA, as the G^2 does not take the number of parameters or the model size into account. Table 3 shows that three models have the best performance regarding the information criteria: The mental model theory with directionality and exclusive guessing at MT (MMTd-Guess2), the mental model theory with exclusive guessing at DA and AC (MMTd-Guess34) and the mental logic with exclusive guessing at DA and AC(as in the original theory by Rips, ML-Guess34). The selection values with these pure guessing trees are much better compared to the original versions. This indicates that theoretical accounts on DA and AC may have to be revised.

Table 3: Results of the MPT fits to the aggregated data set of indicative conditionals by replacing reasoning by guessing.

Model	No. of parameters	G^2	FIA	AIC	BIC
MMTd-Guess2	3	1.6	12	8	29
MMT-Guess34	3	1.6	12	8	29
ML-Guess34	3	1.6	12	8	29
IND-Guess2	4	0	*	8	37
SUP-Guess2	4	0	15	8	37

Note. SUP = suppositional theory; MMTd = mental model theory with directionality; MMT = mental model theory; IND = independence model. Guess2 = MT replaced by the guessing tree; Guess34 = AC and DA with guessing tree only. * FIA cannot be computed for non-binary MPTs.

Counterfactual conditional reasoning

For the third analysis, we tested the performance of the MPT trees of the original theories for conditional reasoning on the counterfactual data. We constructed other sets of MPT models for the mental model theory (cMMT: without directionality and cMMTd: with directionality) according to the aforementioned account of Byrne, which assumes that people build two initial mental models for counterfactuals. But both versions of mental model theories show similar performance. For the counterfactual data, however, the best models are now the independence model with exclusively guessing at the modus tollens (cf. Table 4).

General discussion

While almost all cognitive theories of reasoning aim at explaining human reasoning with conditionals, systematic comparisons are rare. We implemented different theories as multinomial process trees and systematically extended each of the theories with guessing trees and evaluated the goodness-of-fit of (i) the original theories, (ii) the extended models by systematically replacing reasoning subtrees by guessing trees for one to three of the MT, DA, AC-patterns, and (iii) models on

Table 4: Results of the multinomial model fit to the aggregated counterfactual data

Model	No. of parameters	G^2	FIA	AIC	BIC
IND-Guess2	4	0	*	8	31
SUP-Guess2	4	0	12	8	31
MMTd-Guess2	3	6.5	12	13	30
cMMTd-Guess2	3	6.5	12	13	30
ML-Guess34	3	6.5	12	13	30

Note. Models are ordered for the information criteria AIC, BIC, and FIA. Guess2 = MT with guessing tree only; Guess34 = AC and DA with guessing tree only.

counterfactual theories. We performed additionally a literature search and found a high homogeneity of the data. Most of the reported studies asked the reasoner to hold the conditional and the categorical fact as true. The best fitting theory regarding the information criteria AIC, BIC, and FIA (that penalize additional parameters) in case (i) and (ii) is the mental model theory with directionality. For counterfactuals, the best model is the independence model with the modus tollens replaced by pure guessing. Such a difference can be expected as models that perform well in one domain do not necessarily perform well in another. Secondly, the strength of the Bayesian accounts is to represent the difference in strength between antecedent and consequent, which is rarely reflected in most experiments. Another interesting finding is that when comparing models with reasoning and guessing versus guessing alone, some theories are in fact better to assume that some patterns are in fact guessed. In line with the finding of Oberauer (2006), guessing is a very important part in conditional reasoning. The goodness of fit (wrt. AIC and BIC) improves by replacing parts of the theories by guessing in one or more of the three inference rules, especially for MMT. One phenomenon is that reasoner either guess for both AC and DA or MT alone. This might suggest that the processing of MT inference might not be the same as that of the two invalid inferences, AC and DA. Current reasoning theories underestimate the influence of guessing on participant's responses – especially in reasoning with conditionals.

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