In Proceedings of ICCM - 2007- Eighth International Conference on Cognitive Modeling. 261 - 266. Oxford, UK: Taylor & Francis/Psychology Press.

Information Seeking in Complex Problem Solving

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Abstract

Information seeking behavior in human complex problem solving has rarely been well studied. In this paper we studied the information seeking behavior of eye-movement during human complex problem solving in the case of traveling salesman problem. A new model of human TSP solving is proposed to explain the effect of limited amount of visual working memory on the trade-off between local/global information processing and the human information seeking behavior in complex problem solving.

Introduction

When solving problems, information seeking behavior serves as an interface between the world (external information) and cognition (internal information). Hypotheses have been proposed and argued to explain human information seeking behavior in problem solving (Gray & Fu, 2005; Gray, Sims, & Fu 2006). However, most of previous studies on information seeking behaviors are based on experiments either with relatively simple problems and/or with manifested high cost of information seeking, because natural information seeking behavior is hard to measure in the setting of complex problem solving. A recent study in modeling the behavior of human traveling salesman problem solving (Kong & Schunn, 2006) and advanced eye-tracking technology, however, gave us an opportunity to exam the information seeking behavior of human complex problem solving in the case of the traveling salesman problem solving.

The (Euclidean) traveling salesman problem is to find a path of minimum Euclidean distance between points in a plane, which includes each point exactly once and returns to its starting point. As an NP-hard combinatory optimization problem, the traveling salesman problem (TSP) is believed to be "intractable" in computer science for large inputs as long as exact optimal path is concerned.

Experiment

Participants

Six undergraduate students from University of Pittsburgh participated in the experiment.

Materials and Methods

In this experiment, we used the same set of 20 TSP problems as in the experiment described in Kong and Schunn (2006). Ten of them are real world problems borrowed from TSPLIB (http://elib.zib.de/pub/mp-testdata/tsp/tsplib/tsplib.html) ranging in size from 16 points to 100 points. The remaining ten of them were randomly pre-generated according to a uniform distribution ranging in size from 10 points to 80 points. All participants saw the exact same 20 TSP problems, which allow us to examine how well the models predict the influence of particular TSP problems rather than just general trends for the effect of number of points. The experiment was conducted on a Tobii 1750 eye-tracker with a 17" screen. The resolution was set to 1024*768 pixels. During the experiment, participants were 550 to 650 pixels away from screen as recorded by the eye-tracker, measuring by the corresponding screen size and resolution. Participants were asked to find the shortest path possible by indicating the path with mouse-clicks on the screen. A Matlab program recorded all the click data and the eye-tracker recorded all the eye-movement data. The participants were paid 5\$~20\$ based on their performance.

Results

Optimality of the solution is defined as our measurement of performance. Optimality (OPT) of a solution is calculated as the ratio of the optimal path length over the solution path length. So the optimality is a value smaller or equal to 1. The closer the value is to 1, the better performance the participant makes. As figure 1 shows, all the participants found close to optimal paths for all problems (OPT>0.8, MEAN = 0.95, STD = 0.037).

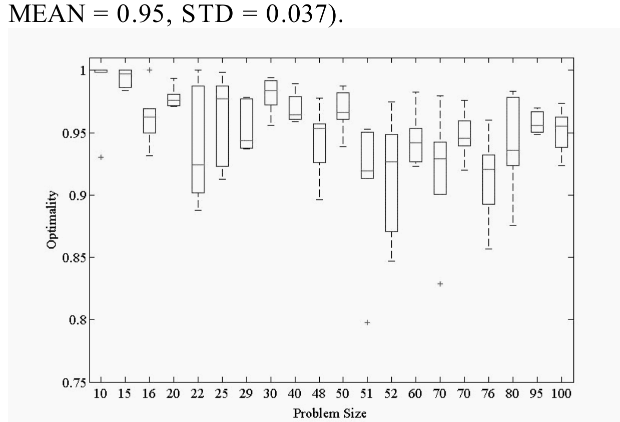

Figure 1: Participants' performance

The eye-movement data recorded by the eye-tracker were used to analyze the information seeking actions. Distance of information seeking of each fixation was defined as the minimum between the following two values:
1. The distance of the fixation to the last visited point
2. The distance of the fixation to the next point to be visited.
The frequency of information seeking follows an exponential distribution along the distance of information seeking ($R^2 = 0.989$).

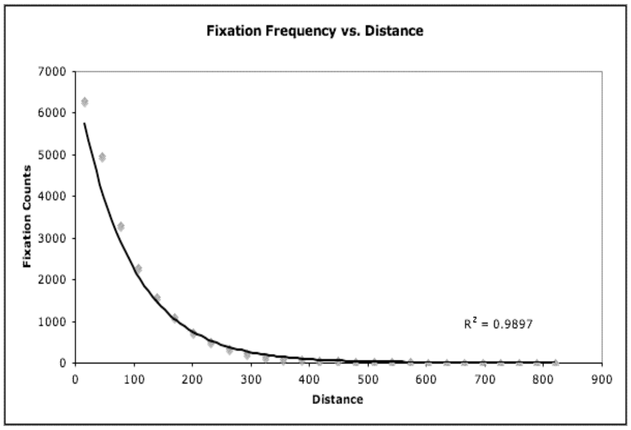

Figure 2: Frequency and distance of information seeking behaviors

We define the global information seeking actions as fixations whose information seeking distances are greater than 200 pixels (about 18 degree of eye-movement in this experiment setting). As an opposite to what was reported in Best (2005), only 23 percent of all the global information seeking actions were made in the beginning of the each trial before 10 percent of points were connected. The rest global fixations distributed through the entire problem solving procedure as shown in figure 3.

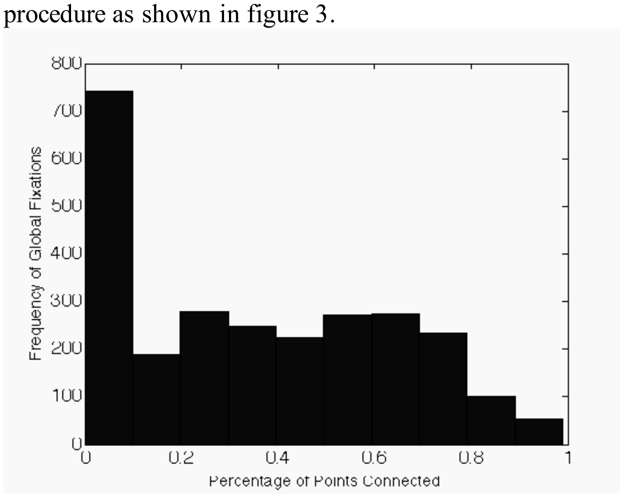

Figure 3: Number of global fixations in each stage of problem solving when part of the points had been connected

Discussion

It is not very surprising that global information seeking actions are biased toward the beginning in the experiment described in Best (2005), since the cost of information seeking changed from low (eye-movement) to high (mouse-movement) when the experiment stage transits. This could be well explained by the soft-constraint hypothesis (Gray & Fu, 2004). But in our experiment, the cost of the information seeking had been low (eye-movement) through the entire problem solving procedure, which we would argue to be a more natural experiment setting. However, the exponential distribution of the frequency vs. distance of information seeking may not be easily explained by the soft-constraint hypothesis as a tradeoff between information seeking cost and its utility. First, the costs (measured by time) by eye-movements of different distances are not significantly different. Second, the utility of information is hard to define in this scenario, since global and local information must interplay with each other to generate a good TSP solution (Kong & Schunn 2006). Our hypothesis is that the limited size of the visual working memory (VWM) could explain this pattern of information seeking behavior. Our intuition is that people do not seek for more global information than they could actually handle in visual working memory. Since the amount of VWM is limited to several chunks, the exponential pattern of information seeking behavior helps to keep both the necessary global and local information in VWM.

Our model

To support our hypothesis, we built a model to simulate the human TSP solving and the information seeking behavior during the process.

To account for the information seeking behavior of human TSP solving, our VWM-Reference TSP model is based on the following two hypotheses and consists of four steps:

First, the VWM only contains a constant number of chunks, which can be set as a parameter in the model. Second, the model only makes constant (in average) number of fixations near the centroids of clusters when they are generated into the VWM to serve as reference points.

Step 1. Initialization
The current working set includes all points. The current point is set to be the starting point.
Step 2. Information Seeking
Points in the current working set are grouped into K clusters according to the K-Means clustering algorithm, (MacQueen, 1967) where K is the size of the VWM in the first iteration and square root of the number of points in the current working set afterwards. The K-Means Clustering Algorithm clusters N data points into K disjoint subsets Sj containing Nj data points so as to minimize the sum of squares criterion:

$$J = \sum_{j=1}^{K} \sum_{n \in S_j} | x_n - \mu_j |^2$$

where x_n is a vector representing the nth point and μ_j is the geometric centroid of the points in Sj. It is assumed that people are capable of noticing clumps of points relatively quickly and easily with their basic perceptual system. This K-Means clustering algorithm is used to proximate the cluster identification, because it is available in standard programming tools and provides the centroids of the clusters as a standard output.

All the centroids are added into the collection of reference points, which was passed from previous iteration.

We then use a spline-curve to connect the current point and all the reference points to sketch a path in a rough scale. The spline-curve is hypothesized to be a general smooth route through the centroids, which captures a general tendency of a globally sketched path.

Step 3. Identify current cluster and refine local information

All the points in the current working cluster are projected to its nearest points on the spline curve. If the number of points projected onto the part of the spline curve between the current point and the first reference point is more than 2, let the current working set to be this set of points, then go back to step 1 and the next iteration begins. When N is the size of the VWM, only the first N reference points, sorted by their projection order on the spline curve, are passed to the next iteration. The rest of them are discarded.

Step 4. Move and rehearse global information

If the number of points projected between the current point and the next reference point is less than two, move from current point to those points according to the sequence they projected onto the spline curve. Set the current working set to be the points projected onto the part of spline between the first and the second reference points. Discard the first reference point from the VWM.

If the number of reference points in the VWM is less than 2, re-identify clusters at the most global level and bring in those centroids back into the VWM.

Repeat this procedure until the number of unvisited points is less than the size of the VWM. Then find the best path for the rest few points.

Figures 4a-e illustrate the steps of our model when solving a 70-points TSP.

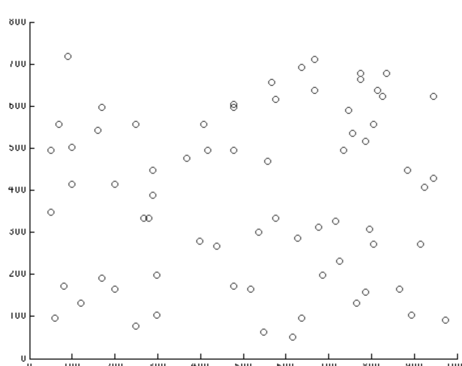

Figure 4a: Original ETSP problem

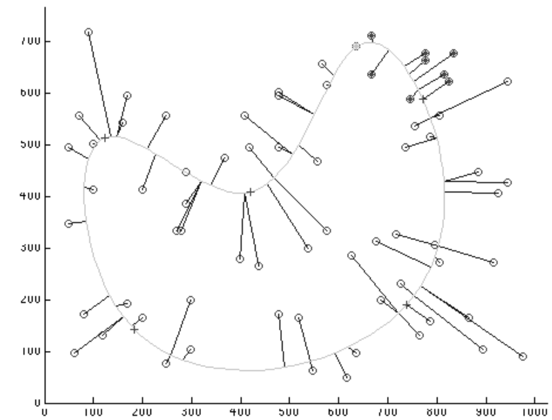

Figure 4b: Seeking global information
'+'s are the locations of the K-Means centroids which are served as reference points in VWM, and the '*' points are those to be served as the current working set in the next iteration

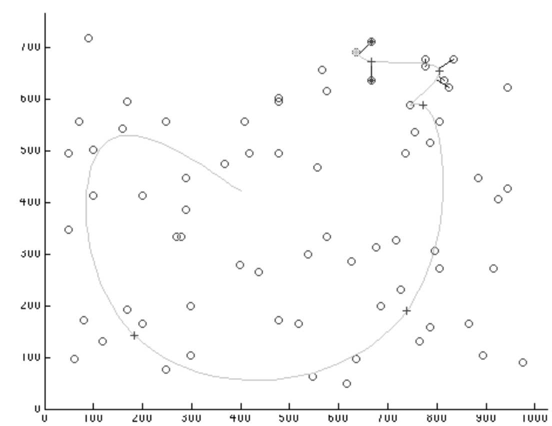

Figure 4c: As the local part of information is refined, some information in global level is discarded. '+'s are the reference points in VWM.

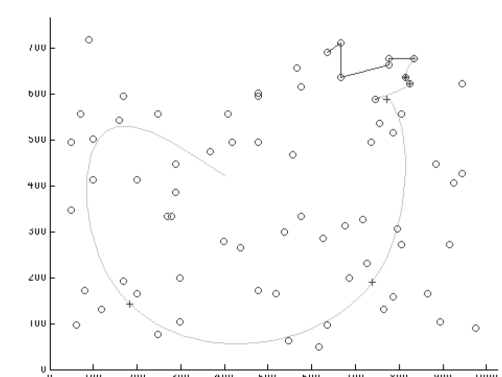

Figure 4d: When there is enough local information, make a move and rehearse the global information

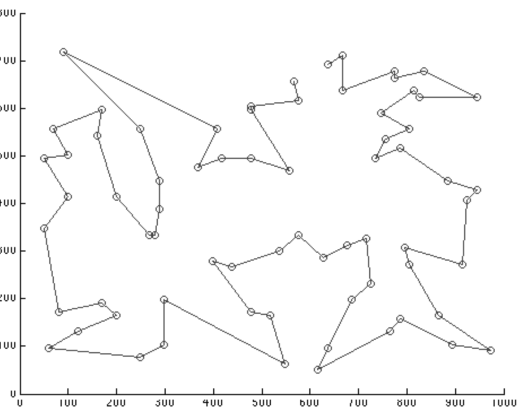

Figure 4e: The final path generated by the model

Model Evaluation and Comparison

Existing models of human TSP solving

Convex Hull

The next simplest model of TSP is the Convex Hull model, which assumes that people compute a traversal around the perimeter points, including inner points opportunistically along the way using a minimal insertion rule. The global information used by this model is the Convex Hull contour, which may be rather complex, and thus require significant working memory. The minimal insertion rule is applied globally at each point during path computation, and points that cause the smallest increase in total path length are inserted. It is somewhat implausible that people would be able to compute these minimal insertions (a local processing task) at a global level.

Sequential Convex Hull Model

MacGregor et al. (2000) adapted the Convex Hull model to a more plausible incremental local search version. This adaptation was base on their finding that humans perform better on problems with fewer interior points within the convex hull (MacGregor & Ormerod, 1996). Second, their experiments provided support for their hypothesis that human participants are sensitive to global information (Ormerod & Chronicle, 1999). We would call this model sequential convex hull model. The outline of the model is as follows (MacGregor et al., 2000):

1. Sketch the connections between adjacent boundary points of the convex hull.
2. Select a starting point and a direction randomly.
3. If the starting point is on the boundary, the starting node is the current node. The arc connecting the current node to the adjacent boundary node in the direction of travel is referred to as the current arc. Proceed to Step 4 immediately. If the starting point is not on the boundary, apply the insertion rule to find the closest arc on the boundary. Connect the starting point to the end node of the closest arc, which is in the direction of travel. This node becomes the current node.
4. Apply the insertion criterion to identify which unconnected interior point is closest to the current arc. Apply the insertion criterion to check whether the closest node is closer to any other arc. If not, proceed to Step 5. If it is, move to the end node of the current arc. This becomes the current node. Repeat Step 4.
5. Insert the closest node. The connection between the current node and the newly inserted node becomes the current arc. Retaining the current node, return to Step 4 and repeat Steps 4 and 5 until a complete tour is obtained.

Pyramid Model

Graham et al.'s model (2000) of traveling salesman problem was inspired by a hierarchical architecture of human visual and spatial perception. Their model first Gaussian-blurs the original set of points into a variety of degrees and stores those blurred images in different layers of hierarchy with the most blurred image on the top. The more blurred images serve as the global information for the less blurred images. Each layer directly guides the next layer below it each time the model develops a node into the path. So layers in the hierarchy change in a repeatedly cascaded process. The Pyramid model computes TSP solutions in the following steps:

1. Gaussian-blur the original n-points TSP image into k-1 different degrees and store them in a k-layer pyramid with the original TSP image on the bottom and the most blurred image on the top.
2. Calculate L_i modes of the image in each layer i. Consider those modes in each layer as nodes in a reduce-sized TSP problem. The top layer has 3 nodes and the bottom layer has n nodes. Layer k has n/b^k nodes. (The parameter b is the reduction ratio. Bottom layer is layer 1.)
3. Layer n (top layer) has 3 nodes and forms a unique tour.
4. Generate a tour of the TSP in each layer by inserting them into the tour on the previously higher layer with the following rules: (a) Sort the intensity level of the mode locations in each layer. (b) Insert these modes into the tour in descending order of their intensity, so as to produce the minimum increase in tour length. Repeat step 4 until the algorithm generates a tour in the bottom layer.

K-Means TSP model

The K-Means TSP model (Kong & Schunn, 2006) is based on the following three steps:

1. Clusters are identified.

In this step, points are grouped according to visual density. Points constructing a higher visual density are more likely to be grouped together. K-Means clustering algorithm was used to generate the $2\times\sqrt{N}$ clusters, where N is the number of points in the problem.

2. A sketch of the path is conceived.

A spline-curve is drawn through all the centroids and back to the start one.

3. Connect all the points along the sketched path.

All the points are projected to the nearest point on the spline-curve. Then we construct the final solution by connecting all the points in the same order as their projection on the spline-curve.

Mean optimality

First we compared the performance of the VWM_TSP model in term of solution optimality with human data and some existing models of human TSP solving including: Convex Hull, Pyramid (Graham, Joshi, & Pizlo 2000; Pizlo, et al. 2006), Kmeans (Kong & Schunn 2006), CHSQ (MacGregor, Ormerod, & Chronicle 2000). We set the size of VWM to be 5 chunks in our VWM_TSP model for this evaluation of optimality, based on existing VWM theories (Pylyshyn, 1989).

The performance of models and human data were plotted in figure 5. Pearson correlations and average signed errors between models and human data were shown in table 1. The VWM_TSP displayed a fairly good correlation and only generated a small amount of error. Though, CHSQ has a better fit to the performance data. VWM_TSP was built under the constraint that the VWM is constant in size. This constraint made our model more theoretically plausible, where CHSQ could have arbitrarily many chunks (invisible lines in its case) in VWM in the extreme case. (Kong & Schunn 2007)

Table 1: Correlation and average signed error of model fits to human accuracy performance

	VWM_TSP	NN	Convex Hull	Pyramid	Kmeans	CHSQ
Correlation	0.62	0.37	-0.02	0.13	0.24	0.69
Ave Signed Error	-0.02	-0.13	0.01	-0.02	-0.03	0.00

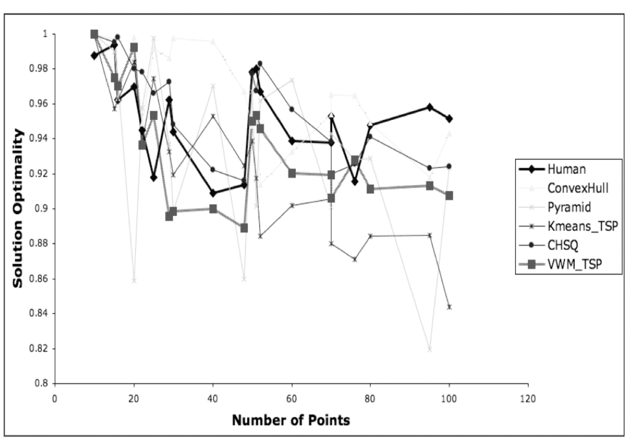

Figure 5: Mean accuracy for models and humans

Number of information seeking actions

Assuming that the VWM_TSP model takes a constant number of information seeking actions around each cluster centroids to generate clusters, we plotted the histogram (# of bins = 30, min = 15 pixs, max = 906 pixs) of VWM_TSP's information seeking distance in figure 6 when VWM size is 5.

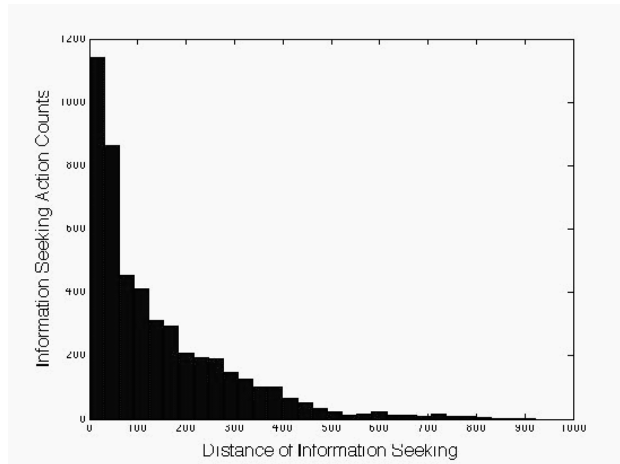

Figure 6: VWM_TSP model's information seek behavior (VWM=5)

As we can see, the count of information seeking actions decreases exponentially with the distance. To study the effect of VWM size on the information seeking behavior, we ran VWM_TSP model with different VWM size parameters (VWMSize = 2,3,4,5,6,7,8,10,15) and plotted the normalized histogram counts of each VWM size as smoothed lines in figure 7. When VWM size is too small (2, 3), the model seeks for global information much more often. In this setting, there wouldn't be much room in VWM to keep global information as soon as local information was developed. So global information had to be re-attended almost on each move. Figure 8 plotted each model setting's R^2 fits to exponential distribution. When VWM size is around 5, the information seeking behavior demonstrated by the model has the best fit to the exponential distribution. When the VWM size is too small or too large, the model's information seeking distance distribution deviates from the exponential distribution. This result is consistent with the existing theories of working memory that the VWM size is around 5 (Pylyshyn, 1989).

To further exam our model, we also looked at how global information seeking behavior varies along time during the problem solving procedure. In figure 9, we plotted the frequency distribution of global information seeking actions during each temporal phase of the problem solving procedure, when part of the points were connected. The model displayed a similar pattern with human data in figure 3, which again supports our hypothesis.

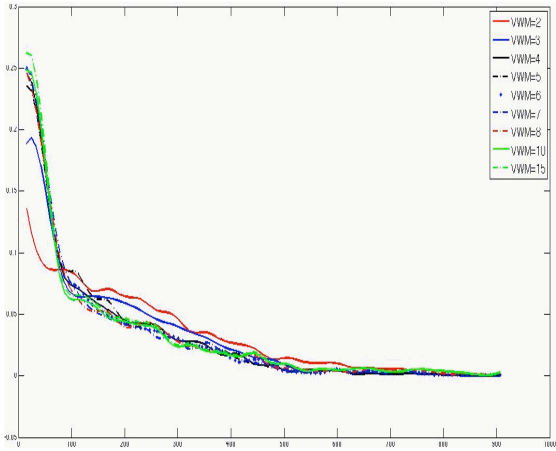

Figure 7: VWM Size vs. Information Seeking Distance Distribution

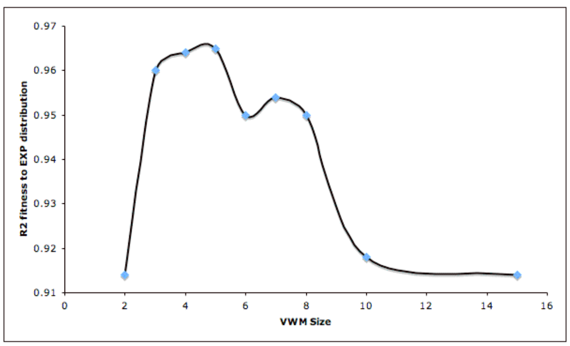

Figure 8: Effect of VWM size on model's fit to exponential distribution

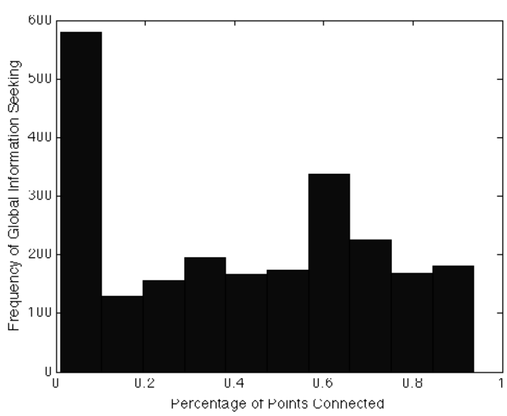

Figure 9: Frequency of global information seeking in each phase of problem solving

Conclusion

The new experimental evidence and simulation results suggested while the cost of information seeking is low and the information utility is hard to define, the limited size of visual working memory plays an important role in the information seeking behavior while solving complex problems. Although the VWM is limited to only several slots, by keeping a good ratio of global information and local information in VWM, human is still capable in solving complex problems to its near optimal solution. Our model while having a good fit to the performance of human TSP solving, also predicts the information seeking behavior during the problem solving procedure. Our model also explored on the question of how different VWM size affects the information seeking behavior during problem solving.

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